ASCHAM SCHOOL

YEAR 12
MATHEMATICS
TRIAL EXAMINATION

2008

Time allowed: 3 hours **Total marks: 120**
Plus 5 minutes reading time

Instructions

1. Attempt all questions.
2. Write in blue or black pen.
3. All questions are of equal value.
4. All necessary working should be shown in each question.
   Marks may be deducted for badly presented work.
5. Standard integrals are provided at the back of the paper.
6. Board- approved calculators may be used.
7. Answer each question in a separate writing booklet.

QUESTION 1

a) Evaluate to 2 decimal places: sin 6 **(2)**

b) Factorise $18x^2 - 2$ **(1)**

c) Solve for x and y:

$$2x - y = 5$$
$$3x + y = 10$$ **(2)**

d) Write as a single fraction in simplest form: $\frac{x}{4} - \frac{x-1}{8}$ **(2)**

e) Rationalise the denominator and write in simplest form:

$$\frac{\sqrt{3}-2}{2\sqrt{3}+1}$$ **(2)**

f) Solve for x : $|x-4| \le 2$ **(2)**

g) For what values of x will the series $1 + (1+x) + (1+x)^2 ....$
have a limiting sum? **(1)**

QUESTION 2

a) Differentiate with respect to x:

i) $x \sin 2x$ **(2)** ii) $\frac{\ln 2x}{x}$ **(2)**

ii) $\cos x^2$ **(2)**

b) Integrate with respect to x:

i) $\sec^2 2x$ **(1)** ii) $\frac{e^{-2x} + e^x}{e^x}$ **(2)**

c) Evaluate: $\int_1^3 \frac{1}{(3x-5)^2} dx$ **(3)**

QUESTION 3

a) On a number plane, plot the approximate positions of A(0,3) B(4,5) and C(8.-3)

i) Show that the equation of AC is $3x+4y-12=0$ **(2)**

ii) Find the perpendicular distance from B to AC. **(1)**

iii) Find the co-ordinates of the point D such that ABCD is a parallelogram. **(1)**

iv) Find the area of parallelogram ABCD **(2)**

b) Solve for x where $0 \le x \le 2\pi$: $2\sin^2 x = \sin x$ **(3)**

c) Find the value(s) of p if the roots of the equation $4x^2 - px + 9 = 0$ are not real. **(3)**

QUESTION 4

a) Find the co-ordinates of the focus and the equation of the directrix of the parabola with equation

$2x + y^2 - 8y = 0$ **(4)**

b) The gradient function of a curve is $2e^{-x} + 3$. If the curve passes through the point (1,4), find the equation of the curve. **(2)**

c) Find n if $\sum_{r=3}^{n}(3r-5) = 5191$ **(4)**

d) In triangle ABC, AB = 5, BC = 4 and angle ABC = 150^0,

find the exact length of AC^2. **(2)**

QUESTION 5

a) Apply Simpson's rule with five function values to find an approximation to $\int_0^{\pi} \sin^2 x dx$ **(3)**

b) A sector OAB of a circle centre O, radius 5m, has area $\frac{25\pi}{24}$ square metres.

i) Find, in degrees, the angle subtended by the arc at the centre. **(2)**

ii) An ant crawling along the arc from A takes 12 minutes to reach B. What is its average speed in millimetres per second? Give your answer to 2 decimal places. **(2)**

c) If the roots of equation $3x^2 - 5x - 7 = 0$ are α and β, find $\alpha^2 + \beta^2$ **(3)**

d) Solve for x if $2\log x = \log 3 + \log(x+6)$ **(2)**

QUESTION 6

a) Consider the function $y = 4\log x + 6 - x$ for $1 \le x \le 10$

i) Find the stationary point and determine its nature. **(4)**

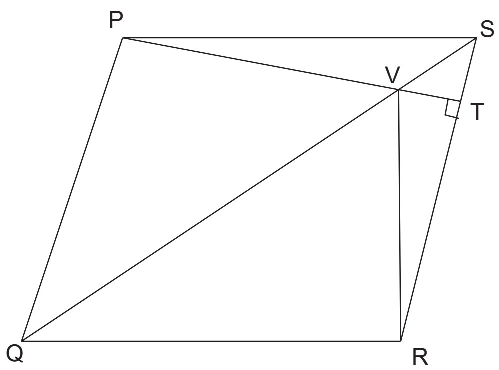
ii) Discuss the concavity of the curve in the given domain. **(2)**

iii) Sketch the curve of $y = 4\log x + 6 - x$ for $1 \le x \le 10$ **(2)**

b) PQRS is a rhombus with PT perpendicular to RS.
Draw the diagram below in your answer books and mark in the data.

i) Show that Δ PQV ≡ Δ RQV **(2)**

ii) Find the size of angle VRQ **(2)**



QUESTION 7

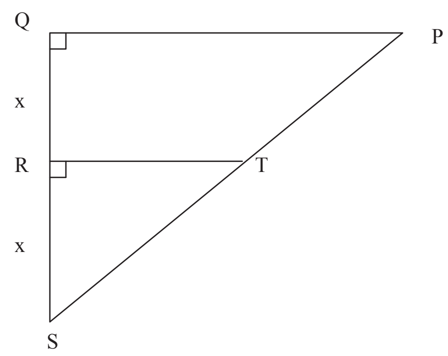
a) Tina wishes to fence off sections of her garden as shown in the diagram below where PS is an existing wall and the lines PQ, TR and QS represent the fencing.
She wishes to have a triangular garden (PQS) of 1600 square metres.

i) Explain why QP = 2RT. **(1)**

ii) Show that the length of fencing L needed is

$$L = 2x + \frac{2400}{x} \text{ metres.}$$ **(2)**

iii) Find the exact value of x which will give the minimum length of fencing Tina needs. **(3)**



b) i) On the same set of axes, draw the graphs of $y = \cos x$ and $y = \sin 2x$ for $0 \le x \le \pi$ **(2)**

ii) The two graphs intersect at $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.
Find the areas bounded fully by the two curves. **(4)**

QUESTION 8

a) Consider the function whose derivative is given by

$\frac{dy}{dx} = x^3(x-2)(x+3)$. Which value of x will give a maximum turning point on the graph of the function? Explain. **(3)**

b) Find the exact volume if the region bounded by the curve $y = 3\log x$, the x axis and x = 5, is rotated about the y axis. **(4)**

c) The amount Q grams of a carbon isotope in a dead tree trunk is given by $Q = Q_0 e^{-kt}$ where Q_0 and k are positive constants and time t is measured in years from the death of the tree.

i) Show that Q satisfies the equation $\frac{dQ}{dt} = -kQ$ **(1)**

ii) Show that if the half life of the isotope is 5500 years, then k = $\frac{1}{5500}\ln 2$ **(2)**

iii) For a particular dead tree trunk, the amount of isotope is only 15% of the original amount in the living tree. How long ago did the tree die? Give your answer to the nearest 1000 years. **(2)**

QUESTION 9

a) A particle moves along a straight line so that its distance x in metres from a fixed point O is given by $x = 2 - 3\sin 2t$ where time t is measured in seconds.

i) Where is the particle initially? **(1)**

ii) Show that the velocity $v = -6\cos 2t$ **(1)**

iii) In which direction is the particle travelling initially? Explain. **(2)**

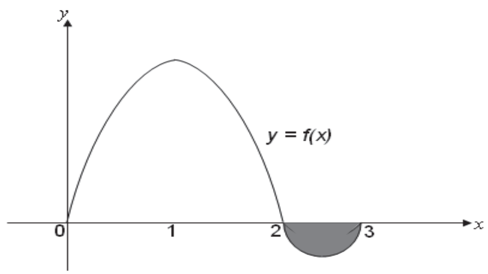
iv) Is the particle speeding up or slowing down when $t = \frac{\pi}{3}$? Explain. **(3)**

v) When does the particle first come to rest? **(1)**

vi) Find the distance travelled by the particle in the first $\frac{\pi}{2}$ seconds. **(2)**

vii) Find the average velocity in the first $\frac{\pi}{4}$ seconds **(1)**

c) Given that $\int_0^2 f(x)\,dx = 11$ and that $\int_0^3 f(x)\,dx = 7$, what is the value of $\int_2^3 f(x)\,dx$. **(1)**



QUESTION 10

a) Alex borrowed $10 000 on 1 January 2008 and agreed to repay it in equal six monthly instalments, the first being paid on 30 June 2008 and the last on 31 December 2018. She is charged 9% per annum interest compounded monthly.
$\$A_n$ represents the amount owing after n months and $M represents the six monthly instalment.

i) Write an expression for the amount Alex owes on 1 July 2008 after she has made her first payment? **(1)**

ii) How many payments does she make altogether? **(1)**

iii) Find the value of M **(3)**

b) A water tank is in the shape of a rectangular prism with a base area of 12m^2 and a height of 2m.
At 12 noon one day, water begins to enter the tank which was dry until then.
The rate at which the volume, V, of water in the tank changes over time t hours is given by
$\frac{dV}{dt}=\frac{24t}{t^2+15}$, where t is time in hours from 12 noon and V is measured in cubic metres.

i) Show that the volume of water in the tank at time t is given by $V=12\ln(\frac{t^2+15}{15})$ **(2)**

ii) At what time would the tank be completely filled with water if the water continued to enter the tank at the given rate. Answer in hours and minutes. **(2)**

iii) At 4pm the water is stopped from entering the tank and the existing water is pumped out. The rate at which the water is pumped out of the tank is given by
$\frac{dV}{dt}=\frac{t^2}{k}$, where k is a constant.

At exactly 6pm the tank is empty.

Find the value of k to 3 decimal places. **(3)**

End of Examination

Solutions

YEAR 12 MATHEMATICS (2UNIT) TRIAL 2008

QUESTION 1

a) $\sin 6 = -0\cdot 28$ (2dp) (2)

b) $18x^2 - 2 = 2(9x^2 - 1)$
$= 2(3x-1)(3x+1)$ (1)

c) $2x - y = 5$ — (1)
$3x + y = 10$ — (2)
$5x = 15$
$x = 3$
Sub in (1) $6 - y = 5$
$y = 1$ (2)

d) $\frac{x}{4} - \frac{x-1}{8} = \frac{2x - x + 1}{8}$
$= \frac{x+1}{8}$ (2)

e) $\frac{\sqrt{3}-2}{2\sqrt{3}+1} = \frac{\sqrt{3}-2}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$
$= \frac{2\times 3 - \sqrt{3} - 4\sqrt{3} + 2}{12 - 1}$
$= \frac{8 - 5\sqrt{3}}{11}$ (2)

f) $|x-4| \le 2$
$-2 \le x - 4 \le 2$
$2 \le x \le 6$ (2)

g) $1 + (1+x) + (1+x)^2 \ldots$
For limiting sum $-1 < r < 1$
$-1 < 1 + x < 1$
$-2 < x < 0$ (1)

QUESTION 2

a) i) $\frac{d}{dx} x \sin 2x$
$= x.2\cos 2x + \sin 2x$
$= 2x\cos 2x + \sin 2x$ (2)

ii) $\frac{d}{dx} \frac{\ln 2x}{x}$
$= \frac{x \frac{1}{2x}.2 - \ln 2x}{x^2}$
$= \frac{1 - \ln 2x}{x^2}$ (2)

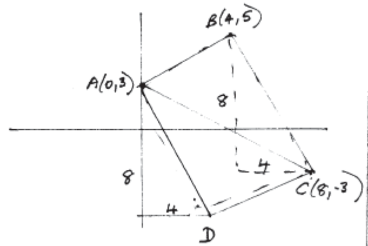
iii) $\frac{d}{dx} \cos x^2 = -2x \sin x^2$

b) i) $\int \sec^2 2x\, dx$
$= \frac{1}{2}\tan 2x + c$

ii) $\int \frac{e^{-2x} + e^x}{e^x}\, dx$
$= \int e^{-3x} + 1 \, dx$
$= \frac{e^{-3x}}{-3} + x + c$
$= -\frac{1}{3}e^{-3x} + x + c$

c) $\int_1^3 \frac{1}{(3x-5)^2}\, dx$
$= \int_1^3 (3x-5)^{-2}\, dx$
$= \left[\frac{(3x-5)^{-1}}{-1\times 3}\right]_1^3$
$= -\frac{1}{3}\left[\frac{1}{3x-5}\right]_1^3$
$= -\frac{1}{3}\left(\frac{1}{9-5} - \frac{1}{3-5}\right)$
$= -\frac{1}{3}\left(\frac{1}{4} + \frac{1}{2}\right)$
$= -\frac{1}{3} \times \frac{3}{4}$
$= -\frac{1}{4}$ (2)

QUESTION 3



i) $m_{AC} = \frac{3+3}{-8}$
$= \frac{6}{-8}$
$= -\frac{3}{4}$ ✓

eqn of AC: $y - 3 = -\frac{3}{4}x$
$4y - 12 = -3x$
$3x + 4y - 12 = 0$ ✓ (2)

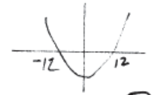
ii) perpendicular distance $= \frac{|4\times 3 + 4\times 5 - 12|}{\sqrt{3^2+4^2}}$
$= \frac{12 + 20 - 12}{5}$
$= \frac{20}{5} = 4$ units (1)

iii) See diagram for gradients
D $(4,-5)$ (1)

iv) Area $\Delta ABC = \frac{1}{2} AC \times$ height
$AC = \sqrt{64+36}$
$= 10$
$\therefore$ Area of parallelogram
$= 2$ Area of ΔABC
$= 2 . \frac{1}{2} . 10 \times 4$
$= 40$ square units (2)

b) $2\sin^2 x = \sin x \quad 0 \le x \le 2\pi$
$2\sin^2 x - \sin x = 0$
$\sin x(2\sin x - 1) = 0$
$\sin x = 0$ or $\sin x = \frac{1}{2}$
$x = 0, \pi, 2\pi$ or $x = \frac{\pi}{6}, \frac{5\pi}{6}$ (3)

c) $4x^2 - px + 9 = 0$
$\Delta = p^2 - 4\times 4 \times 9$
$= p^2 - 144$
For non real roots $\Delta < 0$
$p^2 - 144 < 0$
$(p-12)(p+12) < 0$
$-12 < p < 12$



(3)

QUESTION 4

a) $2x+y^2-8y=0$

$y^2-8y=-2x$

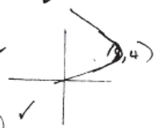
$y^2-8x+16=-2x+16$

$(y-4)^2=-2(x-8)$ ✓

Vertex $=(8,4)$ ½

$4a=2$

$a=\frac{1}{2}$ ½



Focus $=(\frac{1}{2},4)$ ✓

Directrix: $x=\frac{1}{2}$ ✓ (4)

b) $\frac{dy}{dx}=2e^{-x}+3$

$y=-2e^{-x}+3x+c$ ✓

sub $(1,4)$

$4=-2e^{-1}+3+c$

$c=1+2/e$ ✓

∴ equation of curve is

$y=\frac{-2}{e^x}+3x+1+\frac{2}{e}$ (2)

c) $\sum_{r=3}^{n} 3r-5=5191$

$T_1=4$ $T_2=7$ $T_3=10$

$a=4$ $d=3$

$S_N=\frac{N}{2}\{2a+(N-1)d\}$

$5191=\frac{N}{2}\{8+(N-1)3\}$ ✓

$10382=N(8+3N-3)$

$10382=5N+3N^2$

$3N^2+5N-10382=0$ ✓

$N=\frac{-5\pm\sqrt{25+12\times 10382}}{6}$

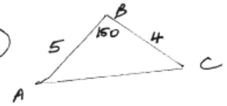
$=58$ or -358 ✓

But $N>0$

∴ $N=58$

∴ $n=60$ ✓ (4)

d)



$AC^2=5^2+4^2-2\times5\times4\times\cos150^\circ$

$=25+16-40\times-\frac{\sqrt{3}}{2}$

$=41+20\sqrt{3}$ (2)

QUESTION 5

a)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\sin^2x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0
	y_0	y_1	y_2	y_3	y_4

$h=\frac{b-a}{n}=\frac{\pi-0}{4}=\frac{\pi}{4}$

$\int_0^{\pi}\sin^2x\,dx\doteq\frac{\frac{\pi}{4}}{3}\{0+0+4(\frac{1}{2}+\frac{1}{2})+2\times1\}$

$\doteq\frac{\pi}{12}(6)$

$\doteq\frac{\pi}{2}$ (3)

b) i) Area of sector $=\frac{1}{2}r^2\theta$

$\frac{25\pi}{24}=\frac{1}{2}25\theta$

$\frac{25\pi}{24}\times\frac{2}{25}=\theta$

$\theta=\frac{\pi}{12}$

$=15^\circ$ (2)

b) ii) Average speed

$=\frac{\text{distance}}{\text{time}}$

$=\frac{r\theta}{12\times60}$

$=\frac{5\times\frac{\pi}{12}\times1000}{12\times60}$

$\doteq1.81805\ldots$ mm/sec

$=1.82$ mm/sec (2dp) (2)

c) $3x^2-5x-7=0$

$\alpha+\beta=\frac{5}{3}$

$\alpha\beta=-\frac{7}{3}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\frac{25}{9}-2\times-\frac{7}{3}$

$=\frac{25}{9}+\frac{14}{3}$

$=7\frac{4}{9}$ (3)

d) $2\log x=\log3+\log(x+6)$

$\log x^2=\log3(x+6)$

$x^2=3x+18$

$x^2-3x-18=0$ ✓

$(x-6)(x+3)=0$

$x=6$ or -3

but $x>0$ ✓

∴ $x=6$ (2)

QUESTION 6

a) $y=4\log x+6-x$, $1\le x\le10$

i) $y'=\frac{4}{x}-1$ ✓

For stationary points $y'=0$

$\frac{4}{x}-1=0$

$4-x=0$

$x=4$ ✓

$y=4\log4+6-4$

$=4\log4+2$ ✓

∴ stationary point is $(4,\ 4\log4+2)$

$y''=-\frac{4}{x^2}$

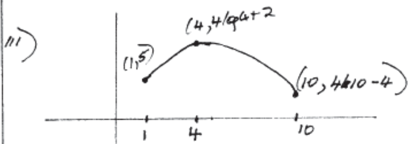
If $x=4$ $y''=\frac{-4}{16}<0$ ∩

∴ $(4,4\log4+2)$ is a maximum turning point. ✓ (4)

ii) $y''=\frac{-4}{x^2}$ which is negative for all x

∴ the curve is always concave down $(x\neq0)$ ✓ (2)

iii)



(2)

$x=1$ $y=4\log1+6-1$

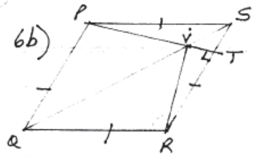
$=5$

$x=10$ $y=4\log10+6-10$

$=4\log10-4$

$\doteq5.2$

6b)



i) In Δs PQV, RQV

$\angle PQV = \angle RQV$ (diagonals bisect angles of rhombus)

$PQ = QR$ (sides of rhombus)

QV is common

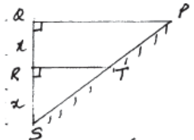
$\therefore \Delta PQV \equiv \Delta RQV$ (SAS) ②

ii) $\angle QPV = 90°$ (cointerior $\angle$ to $\angle VTR$, $RQ \parallel SR$)

$\therefore \angle VRQ = 90°$ (matching $\angle$s of congruent Δs) ②

QUESTION 7

a)



$RT \parallel PQ$ (corresponding $\angle$s =)

i)

$\therefore ST = TP$ (line joining from midpt of 1 side of Δ)

$\therefore RT = \frac{1}{2} QP$ (line joining midpts)

$\therefore QP = 2RT$ ①

ii) Area $= \frac{1}{2} PQ \times 2x$

$1600 = PQ \times x$

$\therefore PQ = \frac{1600}{x}$ ✓

$L = 2x + \frac{1600}{x} + \frac{\frac{1}{2} \times 1600}{x}$

$= 2x + \frac{1600}{x} + \frac{800}{x}$

$= 2x + \frac{2400}{x}$ ✓②

iii) $L' = 2 - \frac{2400}{x^2}$ ✓

For minimum L, $L' = 0$ & $L'' > 0$

$2 = \frac{2400}{x^2}$

$2x^2 = 2400$

$x^2 = 1200$

$x = \pm\sqrt{1200}$

$= \pm 10\sqrt{12}$

$= \pm 20\sqrt{3}$

But $x > 0$ ✓

Test of L'': $L'' = \frac{4800}{x^3}$

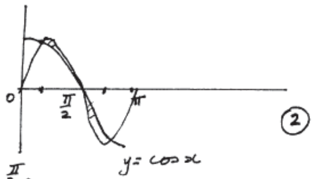
If $x = 20\sqrt{3}$ $L'' = \frac{4800}{(20\sqrt{3})^3}$

> 0 ✓ ③

$\therefore$ if $x = 20\sqrt{3}$

L is minimum length

b) i)



②

$$\text{Area} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x - \cos x \, dx + \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \cos x - \sin 2x \, dx$$

$$= \left[-\frac{\cos 2x}{2} - \sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \left[\sin x + \frac{\cos 2x}{2}\right]_{\frac{\pi}{2}}^{\frac{5\pi}{6}}$$

$$= -\left(\frac{\cos\pi}{2} + \sin\frac{\pi}{2} - \left(\frac{\cos\frac{\pi}{3}}{2} + \sin\frac{\pi}{6}\right)\right) + \left(\sin\frac{5\pi}{6} + \frac{\cos\frac{5\pi}{3}}{2} - \left(\sin\frac{\pi}{2} + \frac{\cos\pi}{2}\right)\right)$$

$$= -\left(-\frac{1}{2} + 1 - \frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{4} - 1 + \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ units}^2$$

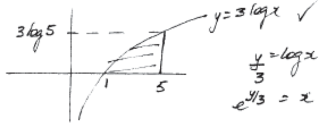
Question 8

a) $\frac{dy}{dx} = x^3(x-2)(x+3)$

x	-4	-3	-1	0	1	2	3
$\frac{dy}{dx}$	-384	0	6	0	-4	0	162

$\therefore$ maximum turning point at $x = 0$ ③

b) $y = 3\log x$



✓

$\frac{y}{3} = \log x$

$e^{y/3} = x$

$$\text{Volume} = \pi \int_0^{3\log 5} 25 - \left(e^{y/3}\right)^2 dy \checkmark$$

$$= \pi\left[25y - \frac{e^{\frac{2y}{3}}}{2/3}\right]_0^{3\log 5} \checkmark$$

$$= \pi\left[75\log 5 - \frac{3}{2}e^{2\log 5} - \left(0 - \frac{3}{2}e^0\right)\right]$$

$$= \pi\left[75\log 5 - \frac{3}{2} \times 25 + \frac{3}{2}\right]$$

$$= \pi\left[75\log 5 - \frac{75}{2} + \frac{3}{2}\right]$$

$$= \pi\left[75\log 5 - \frac{72}{2}\right]$$

$$= \pi(75\log 5 - 36)\ u^3 \checkmark$$

④

c) i) $Q = Q_0 e^{-kt}$

$\frac{dQ}{dt} = Q_0 e^{-kt} \times -k$

$= -kQ$ ①

ii) $t = 5500$ $Q = \frac{1}{2}Q_0$

$\frac{1}{2}Q_0 = Q_0 e^{-k \times 5500}$

$\frac{1}{2} = e^{-5500k}$

$\ln\frac{1}{2} = -5500k$

$k = \frac{-\ln\frac{1}{2}}{5500}$

(or $\frac{\ln 2}{5500}$) ②

iii) If $Q = 0.15 Q_0$

$0.15 Q_0 = Q_0 e^{-kt}$

$0.15 = e^{-\frac{\ln 2}{5500}t}$

$\ln 0.15 = -\frac{\ln 2}{5500} t$

$t = \ln 0.15 \times \frac{5500}{-\ln 2}$

$= 15053.3\ldots\ldots$

$\therefore$ the tree died 15000 years ago (to $\wedge$ 1000 yrs) ②

Question 9

a) $x = 2 - 3\sin 2t$

i) $t=0 \quad x=2$ (1)

$\therefore$ particle is 2m to right of 0

ii) $v = -3\cos 2t \times 2$

$= -6\cos 2t$ (1)

iii) $t=0 \quad v = -6\cos 0$

$= -6 \times 1$

$= -6$

$\therefore$ particle is travelling to the left (2)

iv) $t = \frac{\pi}{3} \quad v = -6 \times \cos\frac{2\pi}{3}$

$= -6 \times -\frac{1}{2}$

$= 3$ ½

$a = 12\sin 2t$ ½

$t = \frac{\pi}{3} \quad a = 12\sin\frac{2\pi}{3}$

$= 12 \times \frac{\sqrt{3}}{2}$

$= 6\sqrt{3}$ ✓

$\therefore$ a + v are > 0 ✓ (3)

$\therefore$ particle is speeding up

v) Particle comes to rest when $v=0$ ie. $-6\cos 2t = 0$

$\cos 2t = 0$

$2t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

$t = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ (1)

$\therefore$ particle first comes to rest after $\frac{\pi}{4}$ seconds

vi) $t=0 \quad x=2$

$t = \frac{\pi}{4} \quad x = 2 - 3\sin 2\times\frac{\pi}{4}$

$= 2 - 3\sin\frac{\pi}{2}$

$= 2 - 3$

$= -1$

$t = \frac{\pi}{2} \quad x = 2 - 3\sin\pi$

$= 2 - 0$

$= 2.$ (2)

$\therefore$ In 1st $\frac{\pi}{2}$ seconds the particle travels $3 + 3 = 6$

vi) Average velocity

$= \frac{-3}{\frac{\pi}{4}}$

$= \frac{-12}{\pi}$ m/s (1)

or $\frac{12}{\pi}$ m/s in negative direction

c) $\int_2^3 f(x)\,dx = 7 - 11$

$= -4$ (1)

Question 10

a) \$10000 1 Jan 2008

6 monthly inst from 30 June 2008

$9\% \text{ pa} = \frac{9}{12}\% \text{ pM} = 0.0075$ | 22 payments, 11 years

i) $A_6 = 10000(1.0075)^6 - M$

ii) 22 payments

iii) $A_{12} = 10000(1.0075)^{12} - M(1 + 1.0075^6)$

⋮

$A_{132} = 10000(1.0075)^{132} - M(1 + 1.0075^6 + 1.0075^{12} + \ldots\ldots 1.0075^{126})$

$= 10000(1.0075)^{132} - M.\frac{(1.0075^{132} - 1)}{1.0075^6 - 1}$

But $A_{132} = 0$

$M = \frac{10000(1.0075)^{132}(1.0075^6 - 1)}{1.0075^{132} - 1}$

$= \$731.24$

PTO. for 10b)

b) i) $\frac{dV}{dt} = \frac{24t}{t^2+15}$

$V = \frac{24}{2}\int \frac{2t}{t^2+15}\,dt$

$= 12\ln(t^2+15) + c$ $\frac{1}{2}$

When $t=0$ $V=0$

$0 = 12\ln(15) + c$

$c = -12\ln 15$ $\frac{1}{2}$

$\therefore V = 12\ln(t^2+15) - 12\ln 15$

$= 12(\ln(t^2+15) - \ln 15)$

$= 12\ln\left(\frac{t^2+15}{15}\right)$ ✓ 2

ii) Volume of tank $= 12 \times 2$

$= 24\,m^3$ ✓

$24 = 12\ln\left(\frac{t^2+15}{15}\right)$

$2 = \ln\left(\frac{t^2+15}{15}\right)$

$e^2 = \frac{t^2+15}{15}$ ✓

$15e^2 = t^2+15$

$t^2 = 15e^2 - 15$

$t^2 = 95.83\ldots$

$t = \pm 9.789\ldots$

$t > 0$

$\therefore$ It would take 9hs 47min ✓

$\therefore$ tank would be full at 21h47. 2

iii) At 4pm $t=4$

$\therefore$ amount of water in tank

$= 12\ln\left(\frac{16+15}{15}\right)$

$= 12\ln\frac{31}{15}\ m^3$. ✓

$\frac{dV}{dt} = \frac{t^2}{k}$

$V = \frac{t^3}{3k} + d$

$t=0$ (ie. at 4pm) $V = 12\ln\frac{31}{15}$ ✓

$\therefore 12\ln\frac{31}{15} = 0 + d$

$\therefore V = \frac{t^3}{3k} + 12\ln\frac{31}{15}$

6pm $t=2$ $V=0$

$0 = \frac{8}{3k} + 12\ln\frac{31}{15}$

$0 = 8 + 36k\ln\frac{31}{15}$

$-8 = 36k\ln\frac{31}{15}$

$k = \frac{-8}{36\ln\frac{31}{15}}$ ✓

$= -0.3061177$

$= -0.306$ (3dp) 3